Boundary of branching random walks on hyperbolic groups

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Abstract

Let be a nonamenable nitely generated in nite hyperbolic group with a symmetric generating set S, and ∂ the hyperbolic boundary of its Cayley graph. Fix a symmetric probability μ on whose support is S, and denote by $\rho = \rho(\mu)$ the spectral radius of the random walk ξ on associated to μ. Let ν be a probability on $\{1, 2, 3, \cdots\}$ with a nite mean λ. Write $\subseteq \partial$ for the boundary of the branching random walk with o spring distribution ν and underlying random walk ξ, and $h(\nu)$ for the Hausdor dimension of . When $\lambda > 1/\rho$, the branching random walk is recurrent, trivially

$$= \partial \ , \ h(\nu) = \dim(\partial \ ).$$

In this talk, we focus on the transient setting i.e. $\lambda \in [1, 1/\rho]$, and prove the following results: $h(\nu)$ is a deterministic function of λ and thus denote it by $h(\lambda)$; and $h(\lambda)$ is continuous and strictly increasing in $\lambda \in [1, 1/\rho]$ and $h(1/\rho) \leq \frac{1}{2} \dim(\partial \ )$; and there is a positive constant C such that

$$h(1/\rho) - h(\lambda) \sim C \sqrt{1/\rho - \lambda} \text{ as } \lambda \uparrow 1/\rho.$$

The above results con rm a conjecture of S. Lalley in his ICM 2006 Lecture (the critical exponent of Hausdor dimensions of boundaries of branching random walks on hyperbolic groups is $1/2$).

This talk is based on a joint work with Shi Zhan, Sidoravicius Vladas and Wang Longmin.